A Review of Droplet Dynamics and Vaporization Modeling for Engineering Calculations

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The present paper reviews the methodologies for representing the droplet motion and vaporization history in two-phase flow computations. The focus is on the use of droplet models that are realistic in terms of their efficient implementation in comprehensive spray simulations, representation of important physical processes, and applicability under a broad range of conditions. The methodologies available at present to simulate droplet motion in complex two-phase flows may be broadly classified into two categories. First one is based on the modified BBO equation. This approach is more comprehensive, but requires modifications and/or correlations at higher droplet Reynolds number. The second approach aims at developing correlations, using detailed numerical simulations or laboratory experiments, for the effects of flow nonuniformity and droplet relative acceleration on the instantaneous drag and lift coefficients. Recent advances made in the droplet vaporization models are also discussed. The advanced vaporization models include the effects of transient liquid heating, gas-phase convection, and variable thermophysical properties. All of these models are discussed, and recommendations are made for their inclusion in comprehensive two-phase computations.

Introduction

Modeling of droplet dynamics and vaporization history in a turbulent, two-phase flow is a challenging problem. First of all, it requires an accurate representation of droplet motion in an unsteady, turbulent flow field. Important considerations here include the description of instantaneous turbulent velocity field for the carrier fluid, and the effects arising from flow nonuniformity and droplet relative acceleration, mass transfer, curvilinear trajectory, and shear-generated lift forces. Droplet vaporization adds another level of complexity to the modeling problem, due to the effects of transient liquid heating, gas-phase convection, and variable thermophysical properties. The problem is further compounded in situations involving nondilute sprays and high pressures, especially those approaching supercritical values.

In this paper, we review the methodologies available at present to represent the droplet dynamics and vaporization history in turbulent, two-phase flows. The review is not intended to be a comprehensive study of the extensive literature that already exists on the subject, rather to focus on those droplet models that have a realistic representation of relevant physical processes, and can be implemented efficiently in comprehensive spray simulations. Many excellent reviews already exists on the various phenomena related to droplet motion and vaporization. The work on droplet motion is reviewed by Clift and Gauvin (1971), Clift et al. (1978), and Leal (1980). The spray modeling work is reviewed by Williams (1973) and Faeth (1977), and the droplet vaporization models by Law (1982), Sirignano (1983), and Aggarwal et al. (1984). The present review considers both the droplet dynamics and the vaporization aspects, and may be viewed as a supplement to the cited reviews. It should also be mentioned that many other important issues, such as droplet–turbulence interactions, droplet–droplet interactions, and high-pressure phenomena, are not covered in this review. In the following, the droplet dynamics models are reviewed first, followed by a review of vaporization models. Conclusions and recommendations for additional work are presented in the last section.

1 Droplet Dynamics Models for (Low-Pressure) Subcritical Conditions

The study of droplet motion has developed in many different directions due to the varied contexts in which they appear. For sufficiently small droplets, the Reynolds number is in the Stokesian regime and the droplet motion can be estimated in terms of an unsteady Stokes flow theory. Originally Basset (1888), Boussinesq (1885), and Oseen (1927) developed a force expression for a slowly moving, accelerating, rigid sphere in a still fluid:

$$m_d \frac{du_{di}}{dt} = 3\pi\mu_g D_d(u_{gi} - u_{di}) + \frac{1}{2}\cdot\frac{\pi}{6}\cdot D_d^3\rho_g \frac{d}{dt}(u_{gi} - u_{di}) + 6\cdot\frac{D_d^2}{4}\cdot\sqrt{\pi\rho_g\mu_g}\cdot\int_{t_0}^{t}\frac{\frac{d}{dt}(u_{gi} - u_{di})}{\sqrt{t - t'}}\cdot dt' \quad (1)$$

where the terms on the right-hand side are, respectively, the Stokes drag, added-mass term, and Basset history term. Later, Tchen (1947) extended the BBO equation to incorporate the effects of a temporally varying flow field on particle transport. Corrsin and Lumley (1956) modified Tchen's equation to account for spatial nonuniformity of the flow field. Riley (1971) revised Corrsin and Lumley's equation to account properly for the effect of undisturbed flow on the particle motion. Maxey and Riley (1983) pointed out some inconsistencies in the modifications suggested by Tchen (1947) and Corrsin and Lumley (1956), and obtained the modified BBO equation in the following form:

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$$\frac{du_{di}}{dt} = \frac{3}{4}\cdot\frac{\rho_g}{\rho_d}\cdot\frac{C_{Ds}}{D_d} V_r\cdot\left(u_{gi} - u_{di} + \frac{1}{24} D_d^2 \nabla^2 u_{gi}\right) + \frac{\rho_g}{\rho_d}\frac{Du_{gi}}{Dt} + \frac{1}{2}\frac{\rho_g}{\rho_d}\frac{d}{dt}\left(u_{gi} - u_{di} + \frac{1}{40} D_d^2 \nabla^2 u_{gi}\right) + \frac{\rho_g}{\rho_d}\sqrt{\frac{81\nu_g}{\pi D_d^2}}\int_{t_0}^{t}\frac{1}{\sqrt{t-t'}}\frac{d}{dt}\left(u_{gi} - u_{di} + \frac{D_d^2}{24}\nabla^2 u_{gi}\right)dt' + \left(1 - \frac{\rho_g}{\rho_d}\right) g_i \quad (2)$$

The derivative d/dt denotes a time derivative following the moving sphere, and D/Dt the time derivative following a fluid element. The terms on the right-hand side correspond in turn to the effects of viscous Stokes drag, pressure gradient of the undisturbed flow, added mass, Basset history term, and buoyancy.

Equations (1) and (2) have several limitations when they are employed in the computations of turbulent sprays. First of all, they are limited to low Reynolds number flows (Re < 1). In addition, the effects of heat and mass transfer (Stefan flow) on the drag coefficient, and the presence of lift force due to shear, are not accounted for. Also, the consideration of turbulence and the proximity of other droplets require further modifications to the droplet drag equation. Different approaches used to account for some of these effects are discussed in the following.

1.1 Corrections for Higher Reynolds Number. A substantial amount of research (Clift et al., 1978) has found that the effects of flow nonuniformity and droplet relative acceleration on the droplet drag and lift forces can significantly alter the droplet motion. A review of the effects of acceleration is given by Clift et al. (1978) and Clift and Gauvin (1971), and of flow nonuniformity by Clift et al. (1978) and Leal (1980). Hughes and Gililand (1952) and Hjelmfelt and Mockros (1967) predicted that a sphere that falls freely experiences drag higher than that given by the Stokes coefficient as it accelerates to its terminal velocity. Odar and Hamilton (1964) used an experimental study and obtained separate correlations for the effects of added mass term and Basset history term at Reynolds number values up to 62. Tsuji et al. (1990) investigated experimentally the drag on a sphere in a periodically pulsating flow for Reynolds number in the range 8000 < Re < 16,000. Their results show that the drag increases in the accelerating flow and decreases in the decelerating flow. Odar (1968) provided data on the drag of a sphere along a circular path in the Reynolds number range from 6 to 185, which shows that the effects of added mass and history of motion increase for this case, whereas the contribution from steady-state drag remains the same as that in a rectilinear motion. Rivero (1991) and Chang (1992) investigated numerically the time-dependent, axisymmetric flow past a sphere at Re varying between 0 and 100, including both oscillatory flows with zero mean and constantly accelerating or decelerating flows. Their results support the idea of an added-mass effect even when there is flow separation. The added-mass term is attributable to the pressure distribution and the coefficient appears to be $\frac{1}{2}$ under a wide range of conditions. The results of these studies indicate that the drag at intermediate or high Reynolds number increases due to droplet relative acceleration. Contrary to the above, Temkin and Kim (1980) and Temkin and Mehta (1982) obtained the drag by observing the motion of a sphere in a shock tube, and reported the opposite results. Ingebo (1956) reported results showing the same trend.

As is evident from the literature, droplet motion in multiphase flows is not completely understood at moderately high Reynolds number. In particular, there is significant uncertainty about the role of flow nonuniformity and droplet acceleration on droplet motion. The previous studies also indicate that the efforts to represent these effects in the droplet dynamics equation have proceeded along two different directions. One approach has employed further modifications (Odar and Hamilton, 1964) to the BBO equation to represent the effects of higher Reynolds number. In the second approach, different correlations have been proposed to modify the standard drag coefficient for including the effects of flow nonuniformity and droplet acceleration.

Approach 1. This approach employs the modified BBO equation, and incorporates the effect of convective acceleration of the gas surrounding the droplet at higher droplet Reynolds number and accounts for shear lift force.

(a) The modified BBO equation. Odar and Hamilton (1964) used an experimental study and obtained separate correlations for the effects of added mass term and Basset history term at Reynolds number values up to 62. They expressed the total drag force by the use of empirical coefficients C_{Ds}, C_A, and C_H.

$$m_d\frac{du_{di}}{dt} = C_{Ds}\cdot\frac{\pi}{4}D_d^2\cdot\frac{1}{2}\cdot\rho_g V_r(u_{gi} - u_{di}) + C_A\cdot\frac{\pi}{6}\cdot D_d^3\rho_g\frac{d}{dt}(u_{gi} - u_{di}) + C_H\cdot\frac{D_d^2}{4}\cdot\sqrt{\pi\rho_g\mu_g}\cdot\int_{t_0}^{t}\frac{\frac{d}{dt}(u_{gi} - u_{di})}{\sqrt{t-t'}}\cdot dt' + \frac{\pi}{6}D_d^3\rho_g\frac{Du_{gi}}{Dt} \quad (3)$$

Nomenclature

A_c = acceleration factor
m = mass
B_M = mass transfer number
$\dot{m}$ = droplet vaporization rate
B_T = heat transfer number
Nu = Nusselt number
C_A = added-mass drag coefficient
R_d = droplet radius
C_D = drag coefficient
Re = droplet Reynolds number = $D_d V_r/\upsilon_g$
C_{Do} = steady-state drag coefficient
Sc = Schmidt number
C_H = Basset history drag coefficient
T = temperature
C_L = lift coefficient
u_i = velocity component in i direction
C_p = specific heat
V_r = magnitude of relative velocity
D = mass diffusivity
x_i = displacement in i direction
D_d = droplet diameter
Y_F = mass fraction of fuel vapor
d_{ij} = deformation rate tensor
ρ = density
g = gravity
μ = viscosity
K = coefficient of Saffman's lift force
ν = kinematics viscosity
κ = nonuniformity parameter
L = latent heat
L' = effective latent heat

Subscripts

g = gas
$i = 1$ = radial direction
$i = 2$ = axial direction
d = droplet
s = droplet surface
∞ = value at infinity

where C_{Ds}, C_A, and C_H are, respectively, the steady-state, added-mass, and history drag coefficients used to represent the effect of higher Reynolds number. C_{Ds} is defined later in Eq. (27). Based on their measurements, Odar (1966) suggested the following empirical formulas for C_A and C_H:

$$C_A = 1.05 - \frac{0.066}{A_C^2 + 0.12} \quad (4)$$

$$C_H = 2.88 + \frac{3.12}{(1 + A_C)^3} \quad (5)$$

A_C is the acceleration factor defined by

$$A_C = \frac{V_r^2}{D_d} \Big/ \frac{dV_r}{dt} \quad (6)$$

and the droplet mass

$$m_d = \tfrac{1}{6}\pi \cdot D_d^3 \rho_d \quad (7)$$

Odar (1966) confirmed that Eqs. (4) and (5), derived for a simple harmonic motion, are valid for the free fall of a sphere in a viscous fluid.

(b) Shear lift force. Saffman (1965, 1968) studied theoretically the lift on a small sphere in a steady, uniform shear flow and gave the following expression for the shear lift force:

$$F_L = 6.46 \rho \nu^{1/2} R_d^2 (u_{g1} - u_{d1}) \left| \frac{du_{g1}}{dx_2} \right|^{1/2} \operatorname{sign} \left(\frac{du_{g1}}{dx_2} \right) \quad (8)$$

where u_{g1} and u_{d1} are the velocities of the fluid and the particle in the x direction, and du_{g1}/dx_2 is the shear rate of the mean flow. In the derivation, it was assumed that

$$\mathrm{Re}_s = \frac{V_s D_d}{\nu} \ll 1, \quad (V_s = |u_{g1} - u_{d1}|) \quad (9)$$

$$\mathrm{Re}_G = \frac{G D_d^2}{\nu} \ll 1, \quad \left(G = \left| \frac{du_{g1}}{dx_2} \right| \right) \quad (10)$$

$$\mathrm{Re}_G = \frac{\Omega D_d^2}{\nu} \ll 1 \quad (11)$$

and

$$\epsilon = \frac{\mathrm{Re}_G^{1/2}}{\mathrm{Re}_s} \gg 1 \quad (12)$$

where Ω is the rotational speed of the sphere. Equation (8) can be used with confidence only when these conditions are met. However, practical situations arise during the study of droplet motion in turbulent flow that require an expression for the shear lift force at larger droplet Reynolds number, Re_s, when conditions (9)–(12) are no longer met.

McLaughlin (1991) extended Saffman's analysis to find shear lift force for $\mathrm{Re}_s \ll 1$, but arbitrary ϵ, and expressed the lift force coefficient as

$$\frac{C_L}{C_{LSa}} = 0.443 J(\epsilon) \quad (13)$$

Saffman's (1965) result was recovered as $J \to 2.255$ for large

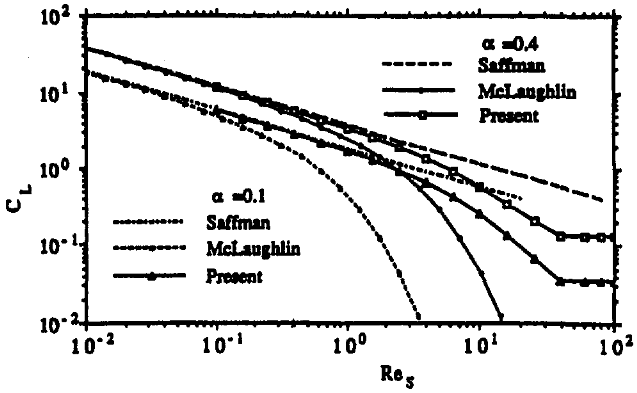


Fig. 1 Shear lift coefficient based on the present approximate expression of Mei (1992) and the analyses of Saffman's (1965) and McLaughlin (1991) (from Mei, 1992)

$J(\epsilon)$. McLaughlin found that $J(\epsilon)$ decreases to zero rapidly as ϵ decreases, which means that Saffman's expression would overpredict the shear lift force. From the table given in McLaughlin (1991), a curve fit for $J(\epsilon)$ is constructed by Mei (1992) for $1 \le \epsilon \le 20$,

$$J(\epsilon) \approx 0.6765[1 + \tanh(2.5 \log_{10} \epsilon + 0.191)] \times [0.667 + \tanh(6(\epsilon - 0.32))] \quad (14)$$

Dandy and Dwyer (1990) reported computational results for the shear lift force at finite $\mathrm{Re}_s (0.1 \le \mathrm{Re}_s \le 100)$ and finite shear rate,

$$\alpha = \frac{G R_d}{V_r} = \frac{1}{2} \mathrm{Re}_s \epsilon^2, \quad (0.005 \le \alpha \le 0.4) \quad (15)$$

After careful examination of the numerical results of Dandy and Dwyer (1990), Mei (1992) proposed the following approximation for C_L:

$$\frac{C_L}{C_{LSa}} = \frac{F_L}{F_{LSa}} = (1 - 0.3314 \alpha^{1/2}) \exp\left(-\frac{\mathrm{Re}_s}{10} \right) + 0.3314 \alpha^{1/2}, \quad \mathrm{Re}_s \le 40$$
$$= 0.0524 (\alpha \, \mathrm{Re}_s)^{1/2}, \quad \mathrm{Re}_s > 40 \quad (16)$$

which combines the analytical result of Saffman (1965) at small Re_s and α and the numerical result of Dandy and Dwyer (1990). In the above, the subscript Sa denotes the corresponding result obtained by Saffman (1968). Figure 1 shows the comparison of the results of Saffman (1965), Dandy and Dwyer (1990) (Eq. (16)), and McLaughlin (1991) (Eqs. (13) and (14)). It can be seen that at low Re_s all three forms agree well with each other for $\alpha = 0.4$. At low shear rate, $\alpha = 0.1$, McLaughlin's result deviates quickly from the numerical result of Dandy and Dwyer as Re_s increases. This is expected because the asymptotic result of McLaughlin is valid only at low Re_s, while a decrease in ϵ means an increase in Re_s for a fixed α. Thus, one cannot expect the result to be accurate for a fixed shear rate α with decreasing ϵ. On the other hand, for a fixed Re_s (say $\mathrm{Re}_s = 0.1$), McLaughlin's analysis indicates that the lift force decreases rapidly as ϵ or α decreases and deviates from Saffman's prediction, while the numerical result of Dandy and Dwyer at $\mathrm{Re}_s = 0.1$ differs only slightly from Saffman's prediction even at $\alpha = 0.005$ and 0.01. It is not clear whether the discrepancy at $\mathrm{Re}_s = 0.1$ and $\alpha \ll 1$ is due to the nonlinear inertia effect neglected in the analysis or to the numerical uncertainties, such as the size of the computational domain and grid resolution, in dealing with three distinctive regions of the flow field defined by Re_s and α.

It should also be mentioned that the analysis of Saffman (1965) and that of McLaughlin (1991) are based on the assumption that $\mathrm{Re}_\Omega \ll 1$. In the low Re regime, the rotation has little effect on the shear lift. The numerical result of Dandy and Dwyer is for $\mathrm{Re}_\Omega = 0$. The effect of rotation on the shear lift force at finite Re_Ω is not clear at present.

Approach 2. The second approach, instead of using the modified BBO equation, employs different correlations to represent the effects of flow nonuniformity and droplet relative acceleration on drag and lift forces operative on a droplet. The effect of flow nonuniformity is represented in terms of a nondimensional parameter κ and the droplet Reynolds number (Puri and Libby, 1990), and the effect of acceleration in terms of a nondimensional parameter A_c (Temkin and Kim, 1980). The correlations are expressed in the following form:

$$C_D = C_{Ds}(1 + f(\kappa, \mathrm{Re})) - C_{AD} \cdot A_C \quad (17)$$

$$C_L = C_{KL} \cdot f(\kappa, \mathrm{Re}) - C_{AL} \cdot A_C \quad (18)$$

where $f(\kappa, \mathrm{Re})$ is a function involving both the flow nonuniformity and Reynolds number.

Then, the droplet acceleration components may be expressed in the radial and axial directions as:

$$\frac{du_{d1}}{dt} = \frac{3}{4} \cdot \frac{\rho_g}{\rho_d} \cdot \frac{V_r}{D_d} [-C_L(u_{g2} - u_{d2}) + C_D(u_{g1} - u_{d1})] \quad (19)$$

$$\frac{du_{d1}}{dt} = \frac{3}{4} \cdot \frac{\rho_g}{\rho_d} \cdot \frac{V_r}{D_d} [C_L(u_{g2} - u_{d2}) + C_D(u_{g1} - u_{d1})] \quad (20)$$

Based on the experimental data, different correlations have been proposed. Temkin and Kim (1980) and Temkin and Mehta (1982) gave the following expression for the drag coefficient:

(*a*) For water droplets, having diameters in the range 87–575 μm, in uniform flow behind a weak shock wave and the Reynolds number range $3.2 < \mathrm{Re} < 77$:

$$C_D = C_{Ds} - K \cdot A_C \quad \text{for} \quad \frac{dU_r}{dt} < 0 \quad (21)$$

where K is a constant of order 1.

(*b*) For water droplets, having diameters in the range of 115–187 μm, in propagating N waves, and the Reynolds number range $9 < \mathrm{Re} < 115$:

$$C_D = C_{Ds} - 0.048 \cdot A_C \quad (-45 < A_C < -3) \quad (22)$$

$$C_D = C_{Ds} - \frac{3.829}{A_C} - 0.204 \quad (5.9 < A_C < 25) \quad (23)$$

The parameter A_C is defined as

$$A_C = \left(\frac{\rho_d}{\rho_g} - 1\right) \cdot \frac{D_d}{V_r^2} \cdot \frac{dV_r}{dt} \quad (24)$$

These relations indicate that acceleration decreases and deceleration increases droplet drag.

Puri and Libby (1990) conducted experiments on droplets moving in a Poiseuille flow in the Reynolds number range of 0.7 to 27 and κ in the range of 10^{-3} to 6×10^{-3} and determined that the droplets experience drag larger than that given by the standard drag. They suggested the following correlation for the drag coefficient:

$$C_D = C_{Ds} \cdot \left(1 + 575\left(\frac{\kappa^2}{\mathrm{Re}}\right)^{3/4}\right) \quad (25)$$

They also determine the presence of lift due to flow nonuniformity. Following the relations of Saffman (1965) and Drew (1978) and considering their own data and those of Eichhorn and Small (1964), they present an empirical formula for the coefficient C_L as

$$C_L = 20 C_{Ds}\left(\frac{\kappa^2}{\mathrm{Re}}\right)^{3/4} \quad (26)$$

For low Reynolds numbers, C_{Ds} is given by the Stokes drag, whereas for high Reynolds number, it involves Stokes drag and a correction such as proposed by Putnam (1961), i.e.,

$$C_{Ds} = \frac{24}{\mathrm{Re}} \cdot \left(1 + \frac{\mathrm{Re}^{2/3}}{6}\right) \quad (27)$$

Peng and Aggarwal (1993) studied the droplet motion in Poiseuille flow and counterflow, and proposed the modified correlations for the effects of flow nonuniformity and relative acceleration at moderately high Reynolds number.

$$C_D = C_{Ds}\left(1 + C_{KD}\left(\frac{\kappa^2}{\mathrm{Re}}\right)^{3/4}\right) - C_{AD} \cdot A_C \quad (28)$$

$$C_L = C_{KL} \cdot C_{Ds}\left(\frac{\kappa^2}{\mathrm{Re}}\right)^{3/4} - C_{AL} \cdot A_C \quad (29)$$

where C_{KD}, C_{AD}, C_{KL} and C_{AL} are constant.

In Poiseuille Flow:

$$C_{AD} = 0.42, \quad C_{AL} = 5 \times 10^{-3} \quad \text{when} \quad A_C < 0.0$$

$$C_{KD} = 575.0, \quad C_{KL} = 50.0$$

In Counterflow:

$$C_{AD} = 0.52, \quad C_{AL} = 0.15 \quad \text{when} \quad A_c < 0.0$$

$$C_{AD} = 0.2, \quad C_{AL} = 0.15 \quad \text{when} \quad A_c > 0.0$$

$$C_{KD} = 725.0, \quad C_{KL} = 400.0$$

Equation (24) is used to calculate A_c.

Figures 2(*a*) and 2(*b*) show the time history of standard drag force, pressure gradient force, added-mass force, Basset history force, and Saffman lift force predicted by using Approach 1 for a droplet moving in Poiseuille flow (taken from Peng and Aggarwal, 1993). The important observation is that the Saffman lift force is significant compared to the standard drag force, and influences the droplet trajectory in the radial direction. However, the other forces such as pressure gradient force, added-mass force, and Basset history force are negligible compared with the standard drag force. Figure 3 shows the comparison of droplet trajectories predicted by Approaches 1 and 2 with experimental data. It is observed that the application of Approach 1 does not give good predictions for this case. For the case of droplet moving in counterflow, the comparison of droplet trajectories predicted by Approaches 1 and 2 is shown

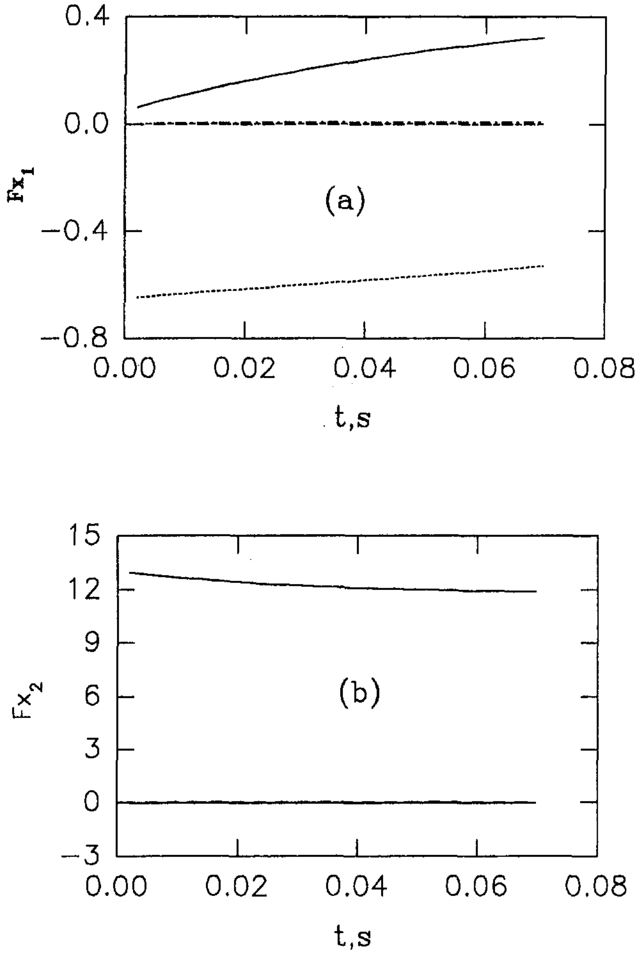


Fig. 2 Time history of individual forces in radial and axial directions for droplet in Poiseuille flow using Approach 1

in Fig. 4. It is indicated that Approach 1 yields trajectory that is significantly different from that obtained experimentally. In order to understand these differences, the time history of various forces represented in Approach 1 is given in Fig. 5. Important observation is that with Approach 1, all the secondary forces

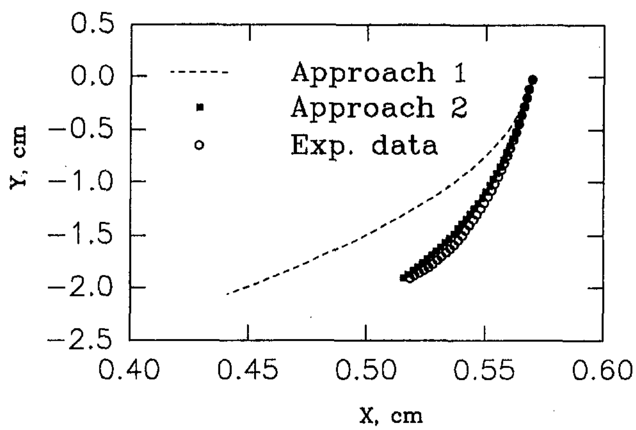


Fig. 3 Comparison of computed and experimental droplet trajectories in Poiseuille flow

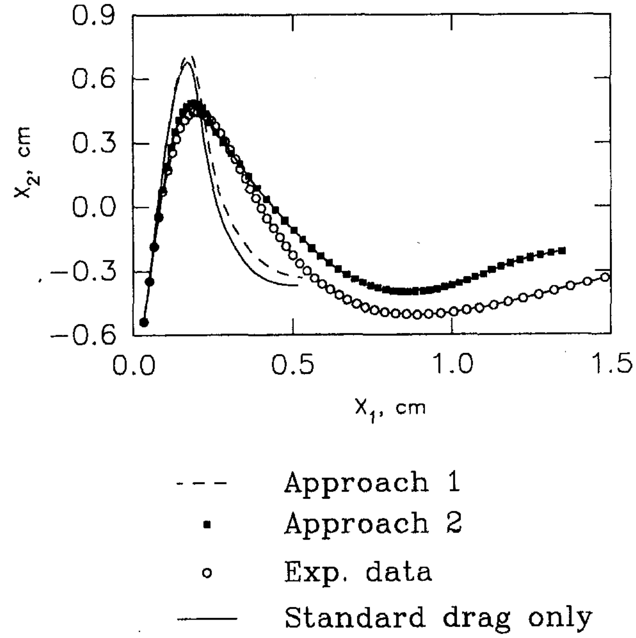


Fig. 4 Comparison of computed and measured droplet trajectories in counterflow

are negligible compared to the standard drag force. Consequently, the predictions of Approach 1 are similar to those obtained by considering only the standard drag.

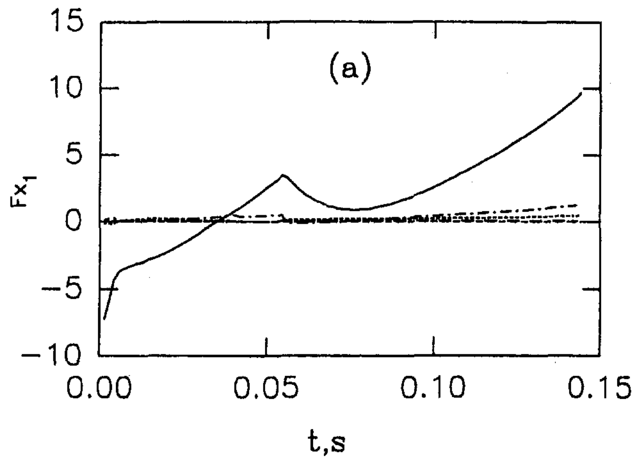
1.2 Correction for Vaporization. Many spray applications involve elevated temperatures where the effects of heat and mass transfer on droplet drag must be considered. Yuen and Chen (1976) found that vaporization affects drag in two ways. First, the temperature and concentration gradients between the droplet surface and the ambient cause substantial reduction in the absolute viscosity of the gas, which decreases friction drag. Second, vaporization affects the boundary layer surrounding the droplet; this blowing effect reduces friction drag and increases form drag. Yuen and Chen argue that the decrease in viscous drag due to blowing is accompanied by an increase in the pressure drag of similar magnitude. In order to account for both variable properties and blowing, a large number of steady-state correlations for drag and heat and mass transfer have been proposed.

Yuen and Chen (1976) showed experimentally that for low to moderate vaporization rates ($B_T = C_p(T_\infty - T_s)/L \ll 3$), the drag coefficient of an evaporating droplet may be approximated by the standard drag curve, provided the gas viscosity μ is evaluated at reference temperature and concentration obtained by using the $\frac{1}{3}$ rule:

$$T_{\text{ref}} = T_s + \tfrac{1}{3}(T_\infty - T_s),$$

$$Y_{F,\text{ref}} = Y_{Fs} + \tfrac{1}{3}(Y_{F\infty} - Y_{Fs}) \qquad (30)$$

Renksizbulut and Yuen (1983a) conducted numerical experiments with droplets in air streams up to 1059 K, and comparing their results with the experimental data of Yuen and Chen (1976) and Eisenklam et al. (1967), proposed the following correlation for the drag coefficient of a droplet evaporating in air:



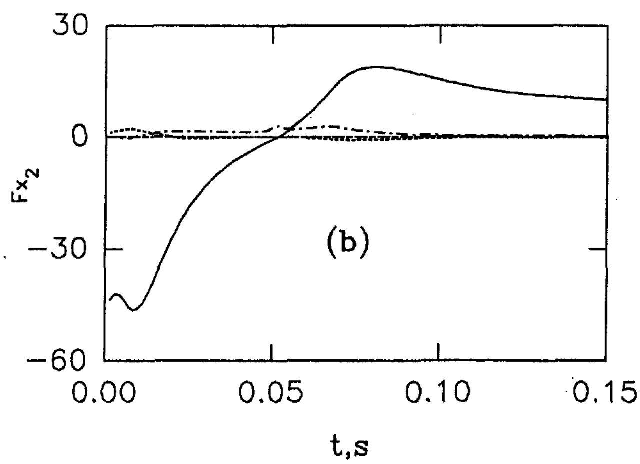


Fig. 5 Time history of individual forces in radial and axial directions for droplet in counterflow using Approach 1

$$C_{Ds} = \frac{24}{\text{Re}} (1 + 0.2\ \text{Re}^{0.63})(1 + B_T)^{-0.2} \tag{31}$$

where the $\frac{1}{2}$ rule is used to evaluate the thermophysical properties, except for the density in Re, which is the free-stream density. Their correlations are valid for $10 < \text{Re} < 300$. The $(1 + B_T)^{0.2}$ factor accounts for the reduction in drag due to the blowing effect of evaporation.

Chiang et al. (1991) proposed the following drag correlation, which agrees within 4 percent with results from their variable-property Navier–Stokes numerical calculation:

$$C_{Ds} = \frac{24}{\text{Re}} (1 + 0.325\ \text{Re}^{0.474})(1 + B_T)^{-0.32} \tag{32}$$

for $0.4 \leq B_T \leq 13$); $30 \leq \text{Re} \leq 200$). The thermophysical properties are evaluated as in the Renksizbulut–Yuen correlation. This correlation indicates a larger reduction in drag due to blowing than does the Renksizbulut–Yuen correlation.

2 Droplet Vaporization Models

The basic droplet vaporization model for an isolated single-component droplet in a stagnant environment was formulated by Godsave (1953), Spalding (1953), Goldsmith and Penner (1954), and Wise et al. (1955). The model has been termed the d^2 law because it predicts that the square of the droplet diameter decreases linearly with time. Since then this model has been studied extensively both experimentally and theoretically. These studies, reviewed by Williams (1973), Faeth (1977), Law (1982), Sirignano (1983), and Aggarwal et al. (1984), consider the effects of relaxing many of the assumptions employed in the basic model. Basically, the existing literature on single-droplet vaporization can be classified into two major categories. In the first category, the basic model is still spherically symmetric but the extensions are proposed to account for the effects of gas-phase convection and transient liquid-phase processes. These include the Ranz–Marshall correlation (1952), infinite conductivity model (Law, 1976), conduction limit model (Law and Sirignano, 1977), the vortex model (Tong and Sirignano, 1983), and the effective-conductivity model (Abramzon and Sirignano, 1989). In the second category, the investigations are based on an axisymmetric model. These include the Prakash–Sirignano model (1978, 1980) and Tong–Sirignano (1983). The present paper aims to complement the extensive reviews by Law (1982), Sirignano (1983), and Aggarwal et al. (1984) and discusses some recent advances that can be easily implemented in turbulent, two-phase flow computations.

(a) Renksizbulut-Yuen Model. Based on experiments, Renksizbulut and Yuen (1983a) provided a correlation for the Nusselt number of a droplet evaporating in air:

$$\text{Nu} = (2 + 0.57\ \text{Re}^{0.5}\ \text{Pr}^{0.333})(1 + B_T)^{-0.7} \tag{33}$$

where the $\frac{1}{2}$ rule is used to evaluate the thermophysical properties, except for the density in the Re, which is the free-stream value. Their correlations are valid for $10 < \text{Re} < 300$. Renksizbulut and Haywood (1988) provided an expression for the vaporization rate as

$$\dot{m} = 2\pi R_d\ \text{Nu} \cdot k_f \frac{(T_s - T_\infty)}{L} \tag{34}$$

where Nu is given by Eq. (37), and k_f and L are thermal conductivity and latent heat of vaporization at T_s, respectively. Equation (38) is obtained by assuming that the droplet is at its wet bulb temperature.

(b) Abramzon–Sirignano Model. Abramzon and Sirignano (1989) provided a new vaporization model of a moving fuel droplet, which represents the extension of the classical droplet vaporization model and includes such important effects as variable physical properties and nonunitary Lewis number in the gas phase, the influence of Stefan flow (blowing) on heat and mass transfer, and the effect of transient liquid heating. In their extended film model, the instantaneous vaporization rate is expressed as:

$$\dot{m} = 2\pi \rho D R_d\ \text{Sh}^* \ln(1 + B_M) \tag{35}$$

and

$$\dot{m} = 2\pi \frac{\kappa}{C_p} R_d\ \text{Nu}^* \ln(1 + B_T) \tag{36}$$

$$H = \frac{C_p(T_\infty - T_s)}{B_T} \tag{37}$$

where Sh* and Nu* are written as:

$$\text{Sh}^* = 2 + (\text{Sh}_0 - 2)/F_M \tag{38}$$

$$\text{Nu}^* = 2 + (\text{Nu}_0 - 2)/F_T \tag{39}$$

The values B_m and B_T are calculated as

$$B_M = \frac{Y_{Fs} - Y_{F\infty}}{1 - Y_{Fs}} \tag{40}$$

$$B_T = (1 - B_M)^{\Phi} - 1 \tag{41}$$

$$\Phi = \frac{C_{pl}}{C_p}\frac{\text{Sh}^*}{\text{Nu}^*}\frac{1}{\text{Le}} \tag{42}$$

The correction factors F_M and F_T are approximated by the function $F(B)$ as

$$F(B) = (1 + B)^{0.7}\frac{\ln(1 + B)}{B} \tag{43}$$

Nu_0 and Sh_0 are the Nusselt and Sherwood numbers employed in the classical model. Two commonly used correlations for Nu_o and Sh_o are Frossling correlations:

$$\text{Nu}_0 = 2 + 0.552\,\text{Re}^{1/2}\,\text{Pr}^{1/3} \tag{44}$$

$$\text{Sh}_0 = 2 + 0.552\,\text{Re}^{1/2}\,\text{Sc}^{1/3} \tag{45}$$

and the Clift et al. (1978) correlation,

$$\text{Nu}_0 = [1 + (1 + \text{Re Pr})^{0.333}f(\text{Re})] \tag{46}$$

$$\text{Sh}_0 = [1 + (1 + \text{Re Sc})^{0.333}f(\text{Re})] \tag{47}$$

where $f(\text{Re}) = 1$ for $\text{Re} \leq 1$ and $f(\text{Re}) = \text{Re}^{0.077}$ for $1 < \text{Re} < 400$ and $0.25 < \text{Sc} < 100$.

All thermophysical properties are evaluated with the $\frac{1}{3}$ rule, except for the density appearing in Re, which is evaluated at the free-stream value. The model agrees with the classical theory in the limit of small Reynolds number, and with the experimental data of Renksizbulut and Yuen (1983b) at high Reynolds numbers.

(c) Chiang–Sirignano Model. From numerical calculations, Chiang et al. (1991) reported Sherwood and Nusselt number correlations similar to those of Renksizbulut and Yuen. They do not provide an expression for the vaporization rate. However, by analogy with the Abramzon–Sirignano model, the vaporization rate is

$$\dot{m} = 4\pi R_d D\,\frac{\text{Sh}}{2}\,B_M \tag{48}$$

where the Sherwood number is

$$\text{Sh} = (2 + 0.46\,\text{Re}^{0.6}\,\text{Sc}^{0.333})(1 + B_M)^{-0.7} \tag{49}$$

valid for $0.3 < B_M < 4.5$ and $30 < \text{Re} < 250$.

3 Discussion and Conclusions

(a) Droplet Dynamics Models. Literature review indicates that both flow nonuniformity and acceleration effects influence the forces on droplets, and there is considerable uncertainty regarding their quantitative contribution to the total drag and lift forces. The effects of curvilinear motion and droplet rotation are also not adequately represented in the droplet dynamics equation. Regarding the quantitative effect of heat and mass transfer on drag, some correlations (Eqs. (30), (31) and (32)) have been proposed, but differences exist in representing the effect of transfer numbers B_M and B_T, and calculating the average properties of gas film surrounding a droplet.

The modified BBO equation (Eq. (2)) is regarded as a more comprehensive approach to model the droplet motion in two-phase flow simulations. The equation is, however, limited to small droplet Reynolds number, $\text{Re} < 1$. Some modifications (Eqs. (4) and (5)) have been proposed, where the effect of higher Reynolds number is represented in terms of correction factors. Following a careful examination of the literature, we write the modified BBO equation in the following form:

$$\begin{aligned}\frac{du_{di}}{dt} &= \frac{3}{4}\cdot\frac{\rho_g}{\rho_d}\cdot\frac{C_{Ds}}{D_d}V_r\cdot(u_{gi} - u_{di}) + \frac{\rho}{\rho_d}\frac{Du_{gi}}{Dt} \\ &+ C_A\frac{1}{2}\frac{\rho_g}{\rho_d}\frac{d}{dt}(u_{gi} - u_{di}) \\ &+ C_H\frac{\rho_g}{\rho_d}\sqrt{\frac{81\nu_g}{\pi D_d^2}}\int_{t_0}^{t}\frac{1}{\sqrt{t - t'}}\frac{d}{dt}(u_{gi} - u_{di})dt' \\ &+ \frac{\rho_g}{\rho_d}\cdot\frac{2K\nu_g^{1/2}d_{ij}}{D_d(d_{lk}d_{kl})^{1/4}}(u_{gj} - u_{dj}) + \left(1 - \frac{\rho_g}{\rho_d}\right)g\end{aligned} \tag{50}$$

where the effect of higher Reynolds number on the unsteady terms is included by using the empirical coefficients C_A and C_H, and on steady-state drag term by using C_{Ds} (see Eq. (27)). In addition, the shear lift force is included, using a generalization of the expression provided by Saffman (1965) for three-dimensional shear fields, with $K = 2.594$ and d_{ij}, the deformation rate tensor, defined as

$$d_{ij} = \tfrac{1}{2}(u_{gij} + u_{gji}) \tag{51}$$

where

$$u_{g_{ij}} = \frac{\partial u_{gi}}{\partial x_j} \tag{52}$$

The generalized lift expressions is restricted to small Reynolds number (Re_s). In addition, it requires that Re_s be smaller than the square root of the Reynolds number based on the velocity gradient, Eq. (12). The modifications such as Eq. (16) could be used when these conditions are not satisfied. Equations (4)–(6) may be used to calculate C_A and C_H. It is clear, however, that more theoretical and experimental investigations are needed in order to develop reliable correction factors that are applicable to a variety of flows. Another important consideration is the relative magnitude of terms in the modified BBO equation representing the flow nonuniformity and acceleration effects. Results from scale analysis and numerical simulations indicate that at high density ratios (ρ_d/ρ_g), representative of liquid fuel sprays at atmospheric pressures, these terms are negligible compared to the steady-state drag term. However, several experimental studies (Temkin and Kim, 1980; Temkin and Mehta, 1982; Puri and Libby, 1989, 1990; Tsuji et al., 1990) find these effects to be significant even at high density ratios, with the implication that the effects are negligible for $\text{Re}_s < 1$, but become important as Re_s increases. At higher pressures, the terms representing these effects in the modified BBO equation become important, but are restricted to $\text{Re}_s < 1$. Moreover, the effects of curvilinear motion and skewness of acceleration vector from velocity vectors on drag and lift are not included. Thus, there exists a need to study the flow nonuniformity and acceleration effects on droplet motion at higher Reynolds number and pressures.

An alternative approach to incorporate the effects of flow nonuniformity and relative acceleration is to use suitable correlations such as Eqs. (28) and (29), with C_{Ds} given by Eq. (27) for nonevaporating droplets, and Eqs. (31) or (32) for evaporating droplet. These correlations are not general, however. They would also be modified by other effects such as the radius of curvature of droplet trajectory and the skewness of acceleration vector from velocity vector. More experimental studies employing simplified configurations, where the relative magnitudes of flow nonuniformity, relative acceleration, and droplet Reynolds number can be independently controlled, are needed in order to develop better correlations.

(b) Vaporization Models. Following the classical d^2 law model, many significant advances in the modeling of droplet vaporization history have been reported. The advances have aimed at relaxing the assumptions employed in the classical treatment. For example, the advanced models now include the effect of variable thermophysical properties, nonunity Lewis number in the gas film outside the droplet, and the effects of transient liquid heating and gas-phase convection. In spite of the availability of such detailed vaporization models, only the simplest ones, based on the d^2 law formulation, have been employed in comprehensive spray computations: only in some recent studies (Aggarwal and Chitre, 1992; Shuen and Chen, 1993) dealing with the computation of turbulent sprays, have more advanced models been introduced. We believe that the computational capabilities are now sufficiently advanced to include the more detailed models in turbulent, two-phase simulations. The simulations should include an algorithm for calculating the variable thermophysical properties, an effective-conductivity model for the effect of transient liquid heating, and the "extended-film" model (Abramzon and Sirignano, 1989) for the effect of gas-phase convection.

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