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Particle Dispersion Enhancement in the Near Region of a Forced Jet

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The present study examines the effects of periodic and random excitations on particle dispersion in the near field of a transitional axisymmetric jet. The study is motivated by the consideration that particle dynamics in the near-jet region is governed by vortex structures, whose behavior, in turn, can be altered through external excitations. A large eddy simulation model based on a fourth-order phase-accurate scheme is employed to simulate the dynamics of vortex rings in an unforced axisymmetric jet, and obtain the dominant frequencies associated with the vortex rings. These frequencies are then used for a periodic forcing of the jet to examine the effects of forcing amplitude and frequency on particle dispersion. A randomly forced jet is also considered to investigate whether exciting all of the dominant frequencies simultaneously can provide greater particle dispersion compared with the single-frequency excitation. Results indicate that external excitation generally leads to higher particle dispersion, with the gain in dispersion increasing with the forcing amplitude. Not only does the particle dispersion exhibit size-selective behavior for both the unforced and the forced jets, but also the dispersion enhancement exhibits a size-dependent behavior, maximizing near a Stokes number of unity. Comparison of randomly forced and periodically forced cases indicates that a single-frequency forcing is more effective in enhancing particle dispersion compared with multiple-frequency forcing. In addition, the preferred mode forcing has the maximum effect on particle dispersion, followed by forcing at the first-pairing and roll-up frequency, respectively. Based on the spectral and flow visualization results, we attribute this behavior to the fact that the preferred-mode forcing makes the second-pairing interaction become more organized and occur closer to the nozzle. This suggests that an effective method of actively controlling particle/droplet distribution in a combustor is to control the attributes of vortex rings, such as their size, frequency, and locations of pairing interactions.

Introduction

LARGE-SCALE structures are known to be an intrinsic part of turbulent shear flows. In particle-laden shear flows, these structures are shown to control the dynamics of particles in the near-flow region. Both laboratory experiments[1,2] and numerical simulations[3,4] indicate that particle dispersion in the near region of a shear layer is controlled by the pairing interactions of these structures, and is characterized by the ratio of particle inertia time to characteristic time associated with large structures. The ratio of the two time scales is defined as the Stokes number St. In the intermediate range of Stokes number St $\sim\mathcal{O}(1)$, the pairing interactions cause particles to disperse significantly more than the corresponding tracer particles. Another aspect, which is of great practical interest, relates to the control of particle dispersion and concentration field by an external perturbation of the initial shear layer. Motivated by the observation that the near field of circular jets is dominated by vortex rings, several experimental studies[5] have examined the behavior of circular jets under controlled excitation. A common finding from these studies is that the dynamics of vortex rings can be significantly modified by external excitation, leading to widely varying jet behavior, such as controlled vortex merging, as well as bifurcating and blooming jets.[6]

The present study examines the effects of acoustic excitations, both periodic and random, on particle dispersion in the near field of a transitional axisymmetric jet. The study is motivated by the consideration of examining active control strategies for achieving optimum performance from spray systems by optimizing the particle/droplet distribution in the system. Because the particle/droplet dynamics in many such applications are controlled by large vortical structures, whose behavior, in turn, can be significantly modified by external excitation, it is naturally of interest to investigate the effects of different excitations on particle dispersion and distribution. A large eddy simulation model based on a fourth-order phase-accurate scheme is employed to investigate the effects of periodic and random forcing on particle dispersion. The model is first used to simulate the dynamics of vortex rings in an unforced axisymmetric jet, and obtain the dominant frequencies associated with the vortex rings. These frequencies are then used for a single-frequency periodic forcing of the jet, and for examining the effects of forcing amplitude and frequency on particle dispersion. A randomly forced jet is also considered to examine whether exciting all of the dominant frequencies simultaneously can provide greater particle dispersion compared with the single-frequency excitation.

Previous works on particle-laden shear flows have mainly focused on particle dynamics in unforced shear layers, examining the effects of large vortex structures on particle dispersion and distribution. Some recent experimental studies[5,7] have considered the effect of external forcing on particle dispersion in shear flows dominated by large vortex structures. Lazaro and Lasheras[2,7] reported experimental results for the dispersion of water droplets in unforced and forced planar shear layers. Based on the measured concentration field and flow visualization, it was concluded that particle dispersion exhibited a monotonic behavior for the unforced case, but a size-selective behavior for the forced case, characterized by the existence of a intermediate particle size range for which the lateral dispersion is maximized. In addition, they observed enhanced par-

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ticle dispersion for the forced case, and a dramatic increase in particle spreading rate when the mixing layer is forced at the first subharmonic frequency. Aggarwal and Xiao[8] used a numerical model to investigate the effects of low-amplitude periodic forcing on particle dynamics in a planar shear layer. While their results were generally in qualitative agreement with the experimental results of Lazaro and Lasheras,[7] they exhibited size-selective dispersion behavior for both unforced and forced shear layers. In addition, consistent with the cited experimental results, it was shown that forcing at the first subharmonic frequency has the most effect on the dispersion of intermediate-sized particles (St ~ 1).

In a recent experimental study, Longmire and Eaton[5] examined the effects of controlled excitation on particle dispersion in the near field of a circular jet. Their results, based on the flow visualization and quantitative mapping of particle number density, clearly demonstrated that both single- and double-frequency forcing can cause significant modification to local particle concentration and dispersion. The present study is strongly motivated by these experimental observations. It differs from a previous numerical study of Aggarwal and Xiao[8] in several aspects. First of all, the dynamics of vortical structures including their pairing interactions and characteristic Strouhal numbers are different in a planar shear layer and an axisymmetric jet. Second, the previous study only considered single-frequency, low-amplitude (less than 1%), periodic excitation for a planar shear layer. In the present investigation, both periodic and random excitations are considered for an axisymmetric jet. In addition, the effects of single- as well as multiple-frequency forcing are investigated to compare the numerical results with the experimental observations of Longmire and Eaton.[5]

Physical–Numerical Model

A schematic sketch of the physical system simulated is given in Fig. 1a, whereas the grid system and computational domain are shown in Fig. 1b. The numerical model for the gas phase is based on the two-dimensional (axisymmetric), time-dependent, inviscid, compressible conservation equations for mass, momentum, and energy density for an ideal gas. The particles' position in the flowfield is computed by solving their equations of motion using a Lagrangian approach. The physical domain of interest, marked by a broken line in Fig. 1a, extends about six jet diameters and is much smaller than the computational domain. The jet velocity U_0 at the inflow boundary is 200 m/s, and the Reynolds number based on the jet diameter of 1.4 cm is 1.87×10^4. A portion of the nozzle is placed inside the computational domain. This allows the pressure at the region in and around the nozzle to respond to fluid accelerations downstream, thus enabling the instabilities to evolve naturally in the calculations.

The dynamics of vortex rings in this axisymmetric jet is simulated by employing a finite difference method based on the monotone integrated large eddy simulation (MILES) algorithm of Boris et al.[9] The algorithm employs a fourth-order accurate, flux-corrected transport method with a directional time-splitting technique with appropriate inflow and outflow boundary conditions, and is shown to reproduce the large-scale features of a variety of flows that are observed in the laboratory experiments. Although no explicit subgrid model is used, the algorithm is demonstrated to incorporate a built-in subgrid model. Further discussion regarding the capability of numerical model is provided in the cited study.[9] Moreover, because the present study considers the influence of large coherent structures on particle dispersion in the initial jet region, the algorithm is expected to be sufficiently accurate for the stated objective. A nonuniform grid is used with 422×122 points in the streamwise and radial direction, respectively. The cells are closely spaced, the minimum spacing is 0.01 cm in the shear layer region immediately following the nozzle exit, and become farther apart as the distance from the nozzle edge increases in the radial and axial directions. The minimum spacing of 0.01 cm in the simulated shear-layer region also gives an effective initial momentum thickness θ_0. This initial thickness gives a reference length scale for describing the jet flow in the initial region. The near-jet region considered in this work is mainly characterized by θ_0, and to a lesser extent also by the jet diameter. Farther downstream, the jet diameter becomes the effective length scale and the dominant mode is the preferred jet mode.[10] The observed jet shear-layer instability frequency $S_\theta = f.\theta_0/U_0 \sim 0.014$, is found to be in the expected range 0.0125–0.0155 of the laboratory experiments.[11] The temporal step size used is 10^{-7} s. In addition, the large-eddy simulation reproduced the dynamics of vortex rings observed in the experiments. This included the Strouhal numbers for the shear layer roll-up and jet-preferred-mode frequencies, the spatial distributions of merging locations, and the shear-layer growth rate. Additional numerical experiments including grid-dependency tests are reported in our earlier study.[4]

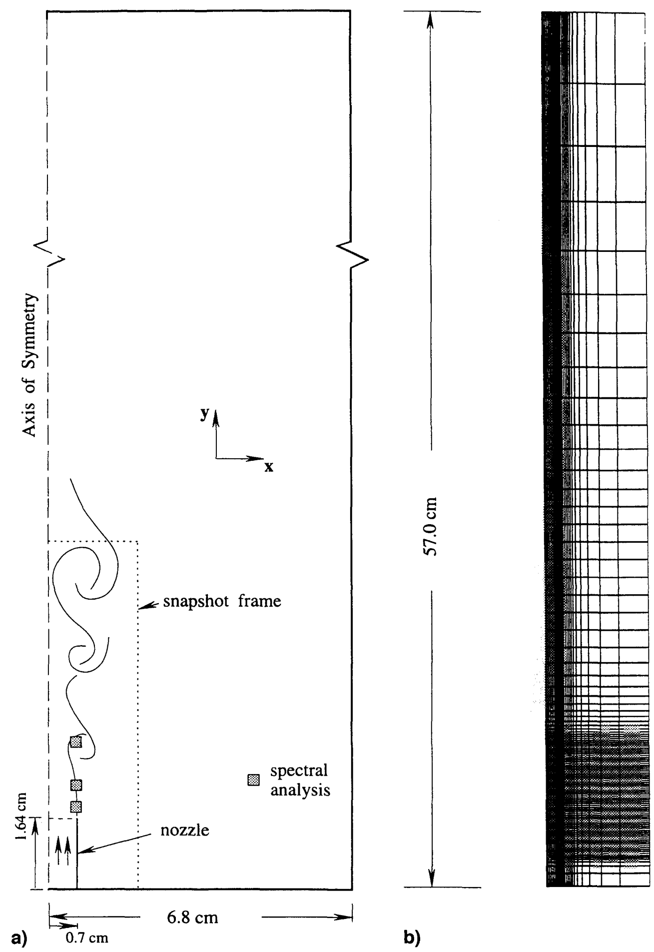

Fig. 1 Schematic of the physical model and the computational domain. Scale a) 1:1 and b) 1:3.

Particles of diameters ranging from 1 to 50 μm are injected from specified radial locations at the nozzle exit, and their dynamics are simulated by solving the standard particle equations using a second-order Runge–Kutta procedure. Results on particle dispersion were established to be independent of the temporal step size and the number of particles used for statistical information. Additional details about the numerical model including the governing equations and boundary conditions are provided in our earlier paper,[4] dealing with particle dispersion in an unforced jet. Standard properties of water are assumed for the particles. For the dispersion results, the near-jet region is considered, extending 9.00 cm (about six jet diameters) in the axial direction.

Results

Numerical simulations were first used to analyze the dynamics of vortex structures and their spectral behavior in an unforced jet. This yielded the dominant frequencies to be used for both single- and double-frequency periodic excitations of the jet. The effects of these excitations on the dynamics of vortex structures and particle dispersion were then investigated. The periodic forcing was simulated by superimposing a sinusoidal fluctuation of specified amplitude and frequency on the jet axial velocity. In addition, a random-forcing case was analyzed to examine whether multiple-frequency forcing would lead to greater particle dispersion compared with single-frequency forcing.

Figure 2 shows the Fourier spectra of gas-phase axial velocity at four different locations in the jet shear layer for the unforced case, obtained by recording velocity history over 40 vortex periods and taking its fast Fourier transform. The maximum error in the Fourier spectra is about 3%. As indicated in Fig. 2, the spectra yield three dominant frequencies of 28, 14, and 7 kHz, corresponding to the shear layer roll-up, and the first and second vortex pairings. The second pairing frequency is commonly referred to as the preferred mode frequency.[10] As discussed in the cited study, the preferred mode is the most dominant of all large-scale coherent structures in an unperturbed circular jet, i.e., an axisymmetric disturbance receives maximum amplification at the preferred mode frequency. The preferred mode Strouhal number obtained from our simulations is 0.49, which is within the range of reported experimental results. It is also in agreement with the experimental value of 0.52, reported by Longmire and Eaton.[1] The Fourier spectra in Fig. 2 further reveals that the dynamics of vortex rings is well organized, with shear-layer rollup occurring at an axial location y = 2.3 cm (about 1.6 jet diameter), and first and second vortex pairings at $y/D \approx 2.2$ and 3.8, respectively. These locations were independently confirmed by the instantaneous isovorticity plots (not shown) at different times.

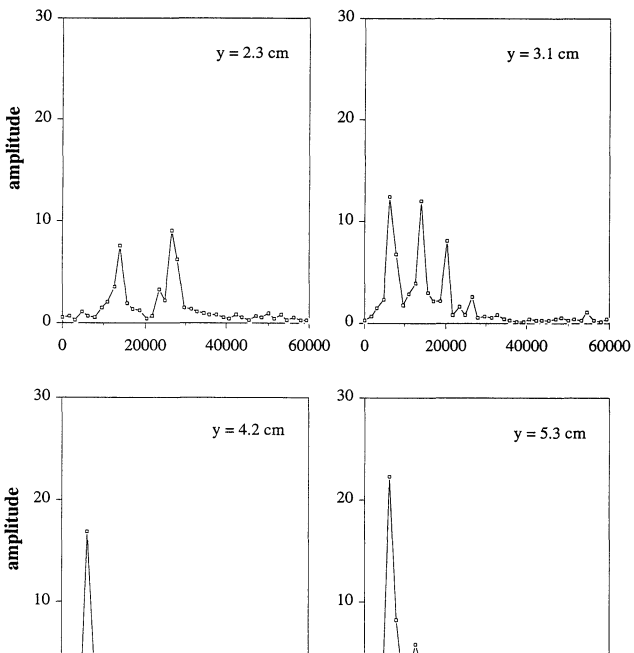

Fig. 2 Fourier spectra of axial velocity, plotted in terms of the amplitude vs the frequency in Hz, at four different axial locations. The radial location is the nozzle rim, r = 0.7 cm.

The Fourier spectra for the corresponding forced cases are depicted in Fig. 3. For the periodically forced cases, a single-frequency forcing is applied at 28, 14, and 7 kHz, corresponding to the rollup, first pairing, and second pairing frequencies of the unforced jet. The corresponding Strouhal numbers ($St_d = f.D/U_j$, D being the jet diameter and U_j the jet velocity) are 1.96, 0.98, and 0.49. The spectral results for each of these forcing frequencies are shown in Figs 3a, 3b, and 3c, respectively, whereas those for random forcing are given in Fig. 3d. The spectra for the periodically forced jet indicates that the preferred mode represents the most amplified mode, which is in accord with the experimental results of Hussain and Zaman.[10] Forcing at the rollup frequency (shear-layer mode) enhances the organization of the roll-up and first pairing, but renders second pairing less organized compared with that for the unforced case. In addition, the comparison of Figs. 2 and 3a indicates that the roll-up and first pairing occur earlier and become more localized compared with the unforced case. With forcing applied at the first pairing frequency, the first and second pairing events appear to become more localized, but the roll-up less localized. In addition, it makes the first pairing to occur closer to the nozzle compared with the unforced case, without significantly changing the locations of roll-up and second pairing. Forcing at the second merging frequency (St_d = 0.49) seems to make all three events more localized. In addition, the first and second pairings occur closer to the nozzle, whereas the roll-up is delayed compared with the unforced case. Results for the randomly forced jet, given in Fig. 3d, indicate that the random forcing is capable of exciting all three modes, although they are less organized compared with the unforced and periodically forced cases.

Results dealing with the effects of external excitation on particle dispersion are depicted in Figs. 4–8. The particle dispersion is quantified in terms of a dispersion function defined as the rms radial displacement of particles from the initial radial injection location:

$$D(t, N) = \left\{ \sum_{i=1}^{N} [x_i(t) - x_{io}]^2/N \right\}^{1/2} \quad (1)$$

where N is the total number of particles in the flowfield at time t, x_i is the radial location of particle i at time t, and x_{io} is the radial injection location of the same particle at nozzle exit. The dispersion function is obtained by simulating a continuous injection of particles in the jet shear layer, and following their individual trajectories in the flow. First, the gas-phase simulation is run until the initial flow transients are out of the domain of interest and the flow has assumed a quasiperiodic character. The particle injection is then started at t = 1.5 ms, which is equivalent to 42 roll-up periods. An injection interval (Δt_{inj}) of 0.001 ms is used for all of the particles that are injected at the jet velocity. Note that the injection interval is much smaller than the smallest vortex time, which is 0.036 ms based on the roll-up frequency. To confirm that the dispersion results are independent of the injection interval, the dispersion function was calculated for Δt_{inj} = 0.001 and 0.0005 ms, and was found to be independent of Δt_{inj}.

Figure 4 shows the temporal variation of dispersion function for different-sized particles injected in a forced jet, subjected to a single-frequency excitation at the roll-up frequency. In the chosen near-jet region (less than six jet diameters), the dispersion increases with time initially, as the particles are convected radially outward by vortex rings during their pairing interactions and more particles are injected into the flow. Eventually, about 1.0 ms after the first injection, the dispersion function levels out, as particles traverse the chosen jet region. This was concluded after simulating continuous injection for a total time of 1.5 ms (42 roll-up periods).

The effect of forcing amplitude on particle dispersion is examined by plotting the dispersion function as a function of particle size for different forcing amplitudes. For this plot,

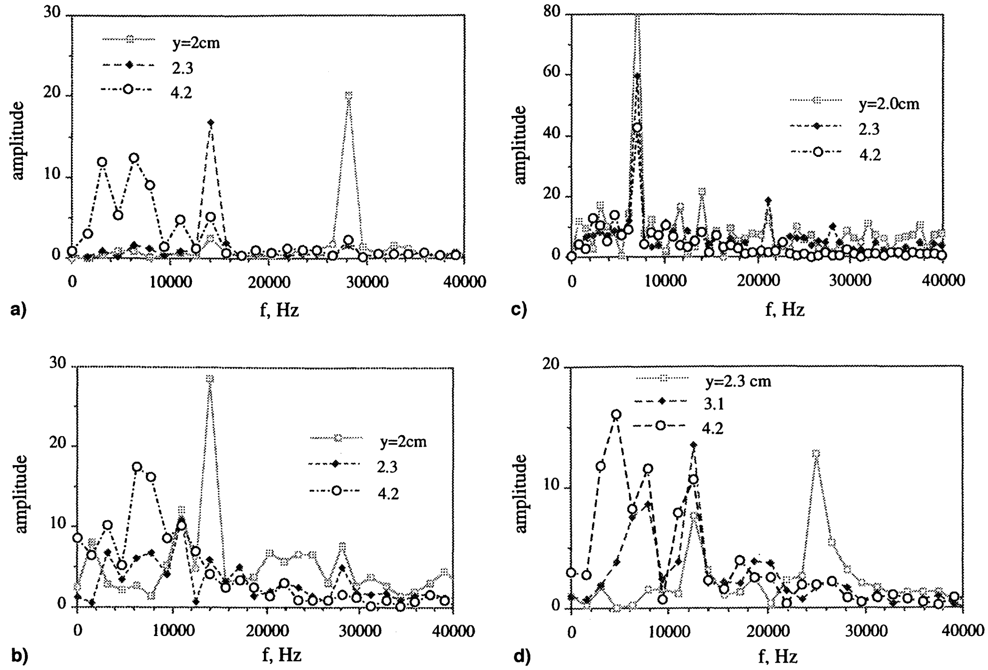

Fig. 3 Fourier spectra of axial velocity at three different axial locations for periodically and randomly forced jets. Periodic forcing frequencies are a) 28, b) 14, and c) 7 kHz, and d) random forcing. The forcing amplitude is 0.1 for all four cases.

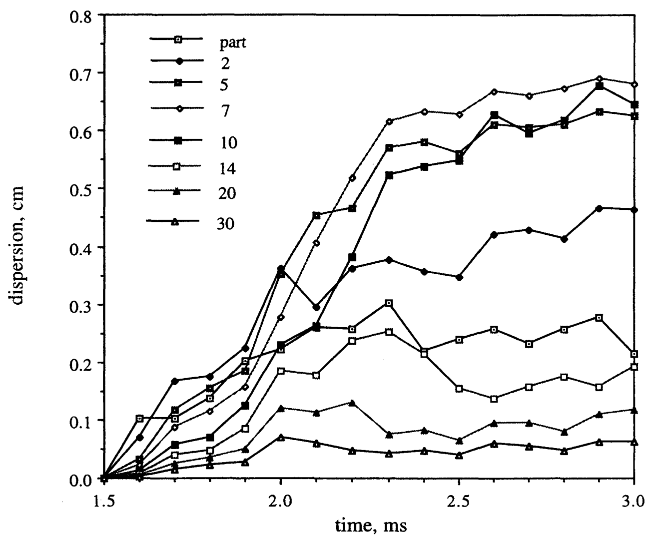

Fig. 4 Dispersion function plotted as a function of time for different particle diameters in a periodically forced jet. The forcing frequency and amplitudes are 28 kHz and 0.1, respectively. "part" refers to the tracer or gas particles, whereas numbers 2, 5, etc., refer to particle diameters in μm.

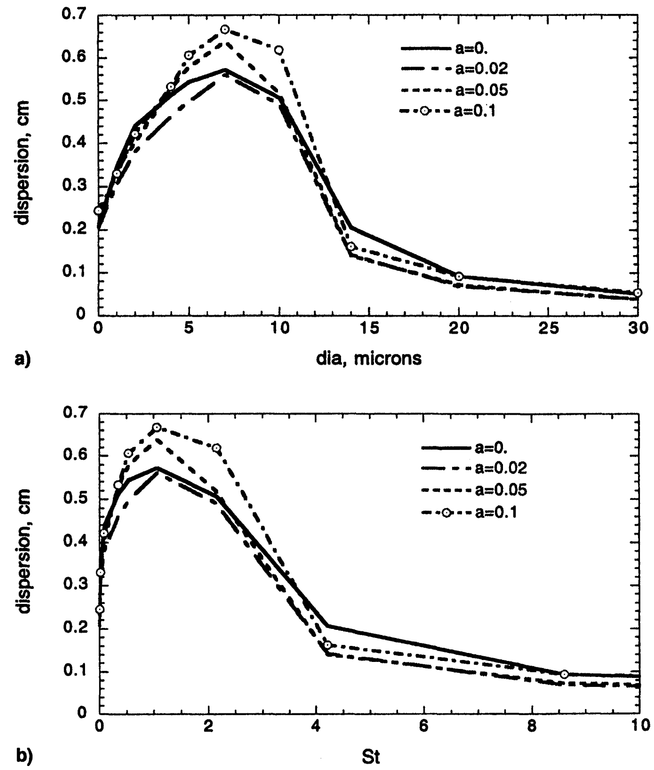

Fig. 5 Effect of forcing amplitude on particle dispersion in a periodically forced jet at the rollup frequency (28 kHz): a) dispersion function vs particle diameter and b) dispersion function vs Stokes number. The case $a = 0$ corresponds to the unforced case, whereas $a = 0.02$, 0.05, and 0.1 indicates a forcing amplitude of 2, 5, and 10%, respectively.

shown in Fig. 5a, an average of the dispersion function values after the initial lapse of 1.0 ms (or 2.5 ms in Fig. 4) is taken for each particle size. Results indicate a typical size-selective dispersion process whereby the intermediate-sized droplets ($\approx$7 μm in the present study) exhibit the maximum dispersion. This behavior has been a subject of numerous experimental and computational studies. An important observation in the context of the present study is that the effect of single-frequency forcing is to enhance the dispersion of intermediate-sized particles, and the effect becomes more pronounced as the forcing amplitude is increased. Because the effect of large vortex structures on particle dispersion is often characterized by a Stokes number St, we replot the results of Fig. 5a in terms of dispersion function vs St for different forcing amplitudes. St is defined here as the ratio of particle response time (t_p), to a characteristic flow time (t_f), the latter based on the preferred mode frequency.[4] As indicated in Fig. 5b, not only does the dispersion process exhibit a size-selective behavior, but also the external excitation has a size-dependent influence on particle dispersion, whereby particles with St near unity experience the most enhancement. This general behavior is observed for all of the forced cases investigated, including the random-forcing case. The dispersion results depicting the effect of forcing amplitude on dispersion for a randomly forced jet are shown in Fig. 6.

The results on the effects of forcing frequency on particle dispersion are summarized in Fig. 7, where we plot the dispersion function vs the Stokes number for three different forcing frequencies. Results for the unforced and randomly forced jets are also included in the figure. The forcing amplitude is 10% ($a = 0.1$) for all the forced cases shown. There are several important observations from this figure.

1) It is shown that periodic forcing at any of the jet characteristic frequencies as well as random forcing can enhance particle dispersion considerably compared with the unforced case.

2) The gain in particle dispersion caused by external excitation appears to be more significant near Stokes number of unity. Although this size-selective response to excitation is an expected result because the particle dispersion for the unforced jet also has a size-selective behavior, maximizing near St $\sim$ 1, an important implication is that external forcing may not necessarily lead to enhanced gas-phase mixing.

3) A periodic forcing at the preferred mode frequency ($St_d = 0.49$) seems to be the most effective mode of excitation for enhancing particle dispersion. Not only does it provide the highest increase in dispersion near St $\sim$ 1, it also leads to higher particle dispersion over a wider range of Stokes numbers, especially for St > 1. To examine this aspect further, we plot the dispersion function vs time for three different Stokes numbers in Fig. 8. Results for three forced cases are compared

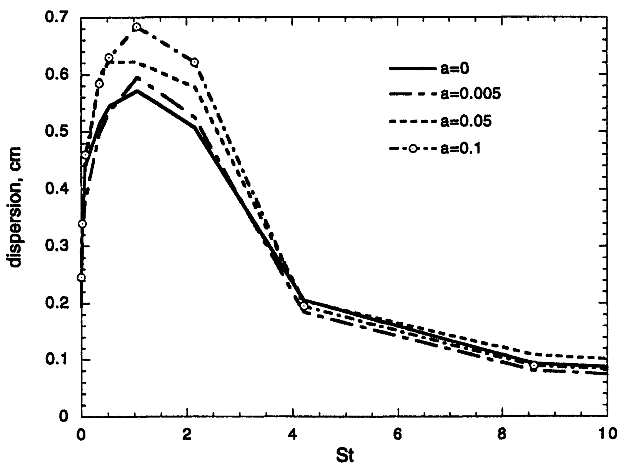

Fig. 6 Dispersion function vs Stokes number for the random-forcing case; effect of forcing amplitude.

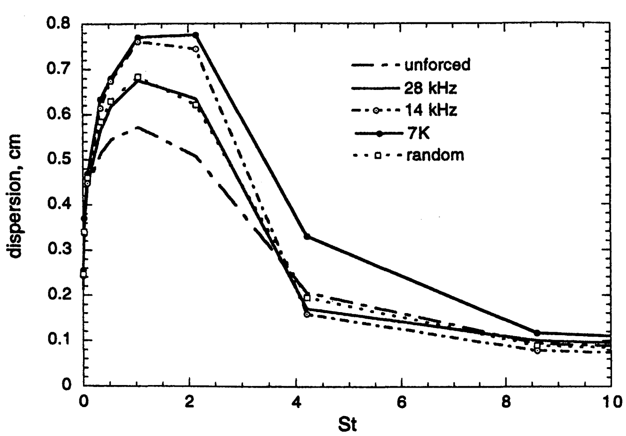

Fig. 7 Dispersion function vs Stokes number for the periodically and randomly forced jets. The periodic forcing is applied at the rollup frequency, first pairing frequency, and second pairing frequency. The forcing amplitude is 10% for all the cases shown.

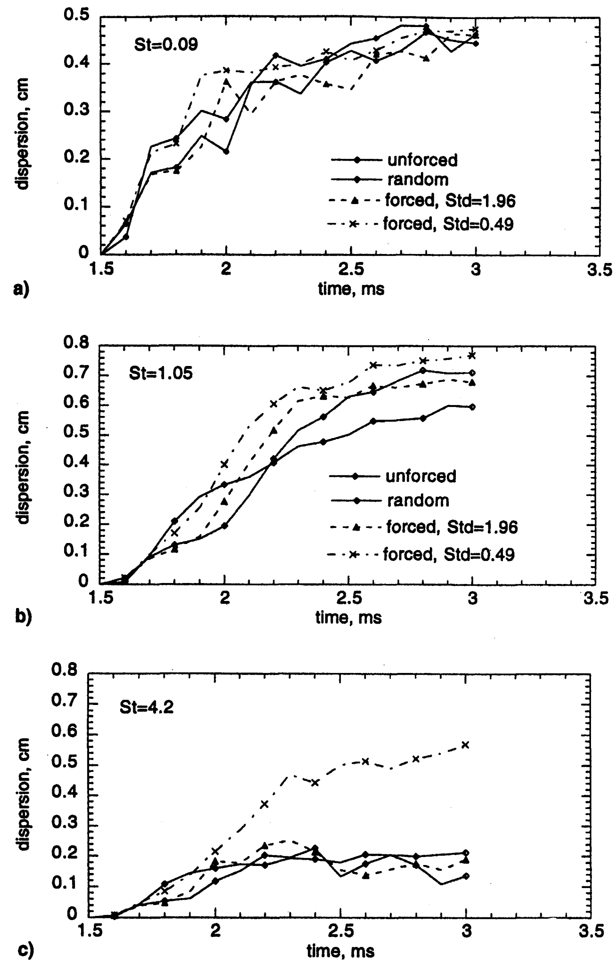

Fig. 8 Dispersion function vs time for three different Stokes numbers. For each St, dispersion function is plotted for the unforced case, randomly forced case, and two cases of single-frequency forcing with forcing Strouhal numbers of 1.96 and 0.49.

with those for the unforced case. The total time corresponds to 42 roll-ups. The forcing amplitude is 10% for all of the cases shown in the figure. An important observation is that while the dispersion of smaller particles ($St << 1.0$) is only slightly affected by external forcing, that of intermediate-sized particles ($St \sim 1$) is significantly enhanced, with the maximum effect provided by forcing at the preferred-mode frequency. In addition, as indicated in Fig. 8c, the preferred-mode forcing can significantly increase the dispersion of larger particles ($St = 4.2$). Detailed flow visualization (not shown) indicated that the preferred-mode forcing makes the second-pairing event become more organized and occur closer to the nozzle exit, a consequence of which is to promote the centrifugal action of vortical structures over a wider range of particle Stokes numbers. As discussed in our earlier study,[4] dealing with an unforced jet, the centrifugal action of vortical structures is responsible for the higher dispersion of intermediate-sized particles, as they get flung out of the spinning ring vortices (during the second pairing interaction), sequentially in order of their size or inertia, i.e., the larger the particle size the earlier it gets flung out. With external excitation applied at the second-pairing frequency, particles with relatively higher Stokes numbers get flung out earlier compared with the cases with no excitation or excitation at other frequencies. Consequently, their dispersion is enhanced.

As a final note, it should be mentioned that the numerical results presented here are in qualitative agreement with the experimental results of Longmire and Eaton.[5] For example, the experimental study used single-frequency forcing at $St_d = 0.9$

and 0.5, and a significant effect of forcing on particle distribution was observed resulting in higher dispersion. In addition, the effect was found to be much stronger with the $St_d = 0.5$ forcing compared with $St_d = 0.9$. The numerical results here exhibit a similar behavior; indicating that the effect of forcing on particle dispersion is maximized at $St_d = 0.49$. It should also be noted that a quantitative comparison was not possible because of different flow conditions and lack of information regarding initial conditions in the two studies. For example, the jet Reynolds number was 1.87×10^4 in the present study, and 1.92×10^4 in the experimental study. Moreover, in the experimental study, the particles were injected with a finite but unknown radial velocity, and their axial injection velocity was not reported. The particle dispersion is significantly influenced by both the axial and radial injection velocity components. The effect of injection condition on particle distribution in the unforced jet has been examined in our earlier study.[4]

Conclusions

In this numerical study, we have investigated the effects of controlled excitation on particle dispersion in the near field of an axisymmetric jet. Both periodic- and random-forcing cases have been analyzed, and the effects of forcing frequency and amplitude have been examined. For the periodic excitation, a single-frequency forcing has been applied to the jet axial velocity, corresponding to the roll-up, first pairing, and second pairing modes of the unforced jet. Important observations are as follows:

1) For all of the cases analyzed, the external excitation leads to higher particle dispersion, with the gain in dispersion increasing with the forcing amplitude.

2) Not only does the particle dispersion exhibit size-selective behavior for both the unforced and the forced jets, but also the dispersion enhancement caused by external forcing shows a size-dependent behavior; i.e., most of the dispersion enhancement occurs near $St \sim \mathcal{O}(1)$. Because the size-selective dispersion process for the unforced case is attributable to the convective effect of vortex rings and their pairing interactions, the implication is that external excitation modifies the vortex ring dynamics in such a way that it leads to higher dispersion. In other words, an effective method of actively controlling particle/droplet distribution in a combustor is to control the attributes of vortex rings, such as their size, frequency, and locations of pairing interactions.

3) For a given amplitude, the single-frequency forcing at the preferred mode ($St_d = 0.49$) has the maximum effect on particle dispersion, followed by forcing at the first-pairing and roll-up frequency, respectively. Based on the spectral results and limited flow visualization, we attribute this behavior to the fact the preferred-mode forcing makes the second-pairing interaction become more organized and occur closer to the nozzle. A random forcing is also found to enhance particle dispersion, although the effect is generally not as strong as that with periodic forcing. Because the random forcing is able to excite all of the dominant natural modes, it is akin to a multiple-frequency forcing, with the implication that single-frequency forcing is more effective in modifying particle dispersion compared with multiple-frequency forcing. It should be noted, however, that by controlling the phases of different frequency components, it may be possible to enhance the effectiveness of multiple-frequency forcing in controlling particle dispersion, as observed by Longmire and Eaton.[5]

4) Numerical results show qualitative agreement with the experimental results of Longmire and Eaton.[5] For example, both the experimental and numerical studies indicate that the single-frequency forcing can significantly modify particle dispersion, with the effect maximizing near $St_d = 0.5$.

5) Finally, it is useful to mention the relevance of this study to the active control strategies that may be used for enhancing the performance of a liquid-fueled combustor. Clearly, the development of analytical/numerical tools that can predict the dominant vortex frequencies and other relevant system frequencies will be directly useful for devising such control strategies. For example, the computational algorithms can provide information about the dominant frequencies that directly influence the combustor instability modes for a wide range of operating conditions. Such algorithms can also be used to predict frequencies that are most effective in modifying the droplet concentration field, and, thereby, the fuel vapor distribution in the combustor.

Acknowledgments

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